1 Domain structures (book ch. 4)

magnetic field H ↑ ← caused by a magnet

magnetic moment M ↑ ← material

$E = -\mu_0 H \cdot M$ $\quad E = +\mu_0 H \cdot M$

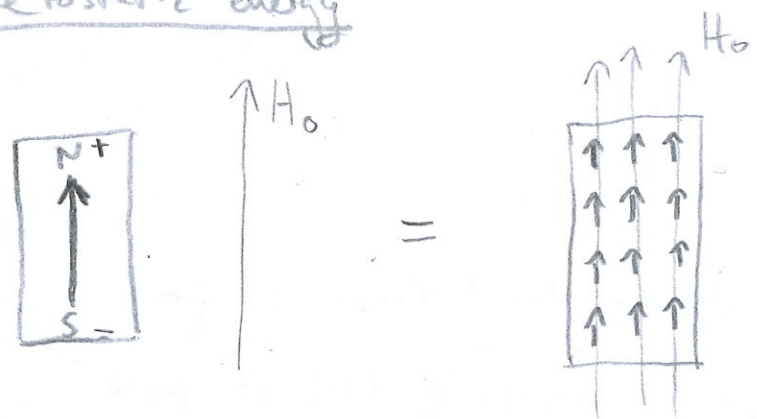
↑ magn. permeability in vacuum

$$\text{Energy} = -\mu_0 \vec{H} \cdot \vec{M}$$

What is the energy of a magnetic grain?

magnetostatic + (self) demagnetizing energy

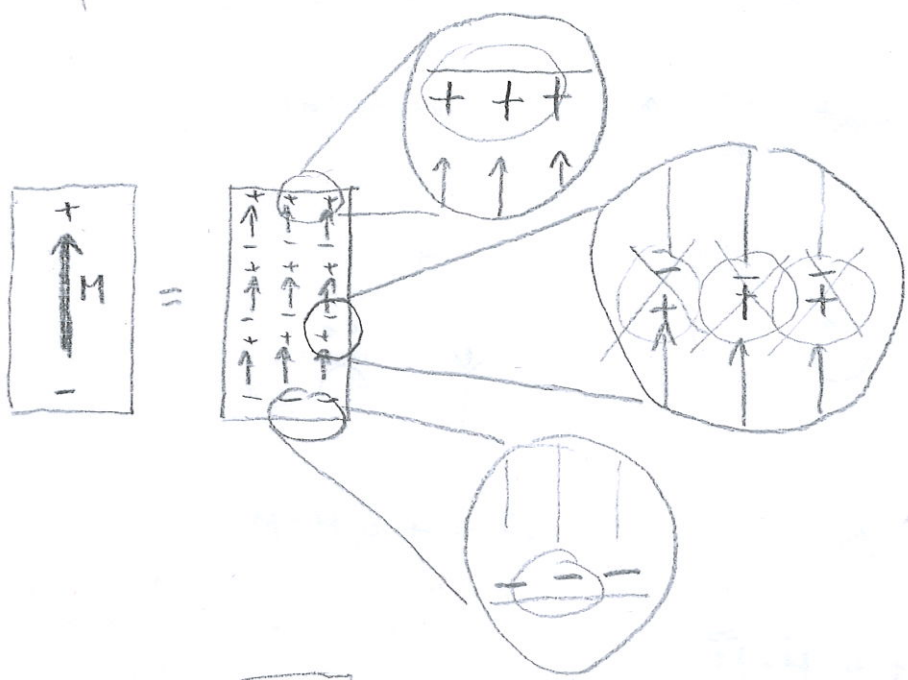
1) Magnetostatic energy

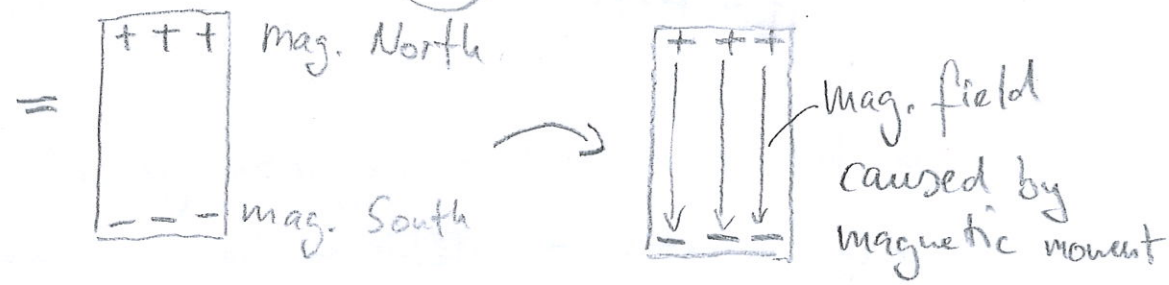


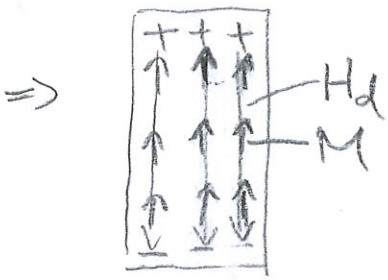
$$E_H = -\int \mu_0 \vec{H}_0 \cdot \vec{M} \, dV = -\mu_0 \vec{H}_0 \cdot \vec{M} V$$

2 (Self) demagnetizing energy

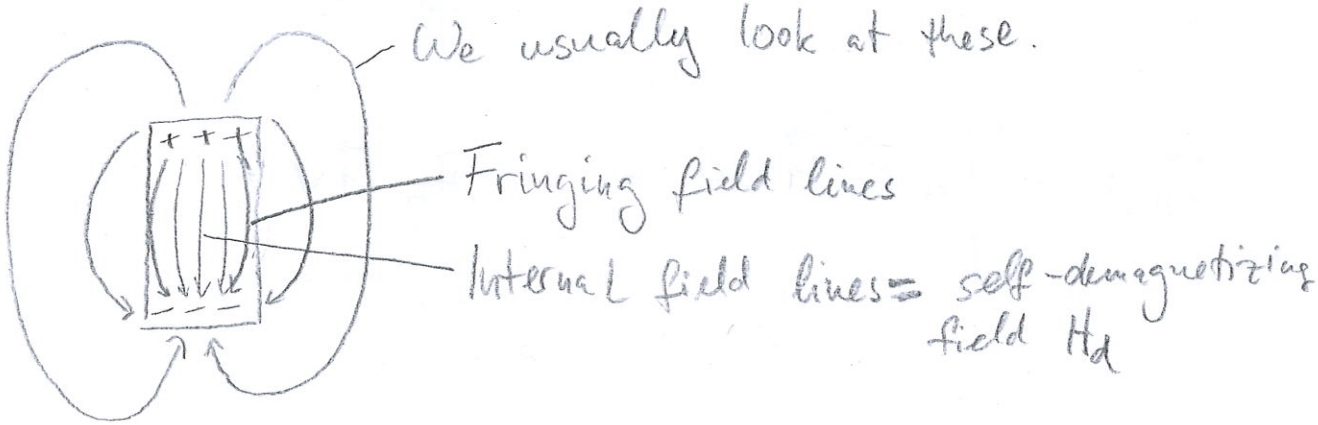
In 0-field:







How exactly do the field lines H_d go?



Approximation

straight field lines H_d

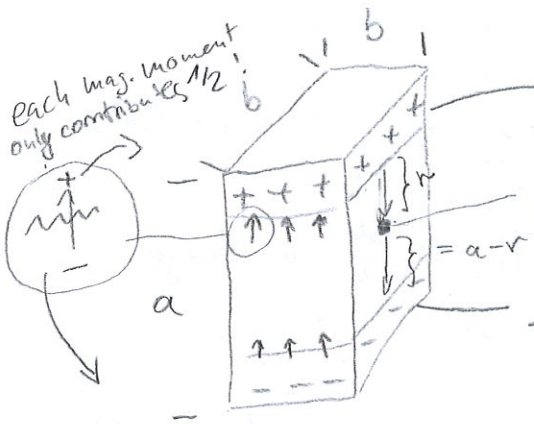
straight magnetic moments M

What is the demagnetizing energy?

$$E_d = -\int \mu_0 M \cdot H_d \, dV$$

↑

?



(per volume)

$$Q_m \propto M \cdot b^2 \cdot \frac{1}{2}$$

$$H_d(r) \propto \frac{Q_m}{r^2}\hat{\vec{r}} - \frac{Q_m}{(a-r)^2}\hat{\vec{r}}$$

$$-Q_m \propto -M \cdot b^2 \cdot \frac{1}{2}$$

$$\Rightarrow H_d(r) = \left(\frac{1}{r^2} - \frac{1}{(a-r)^2}\right) \cdot Q_m = \left(\frac{b^2}{r^2} - \frac{b^2}{(a-r)^2}\right) \cdot \frac{1}{2} \cdot M$$

$$\Rightarrow E_d = -\int \mu_0 \left(\frac{b^2}{r^2} - \frac{b^2}{(a-r)^2}\right) \cdot \frac{1}{2} M^2 \, dV$$

$$= -\frac{1}{2}\mu_0 \underbrace{\int \left(\frac{b^2}{r^2} - \frac{b^2}{(a-r)^2}\right) dV}_{\equiv N} \cdot M^2$$

$$\Rightarrow \boxed{E_d = -\frac{1}{2}\mu_0 N M^2 = -\frac{1}{2}\mu_0 H_K M}$$

N: demagnetizing factor

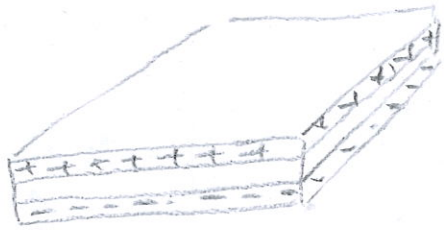
H_K: coercivity

$H_K = N \cdot M$

What is N?

a) 

needle (prolate / elongate)

b) 

disc (oblate)

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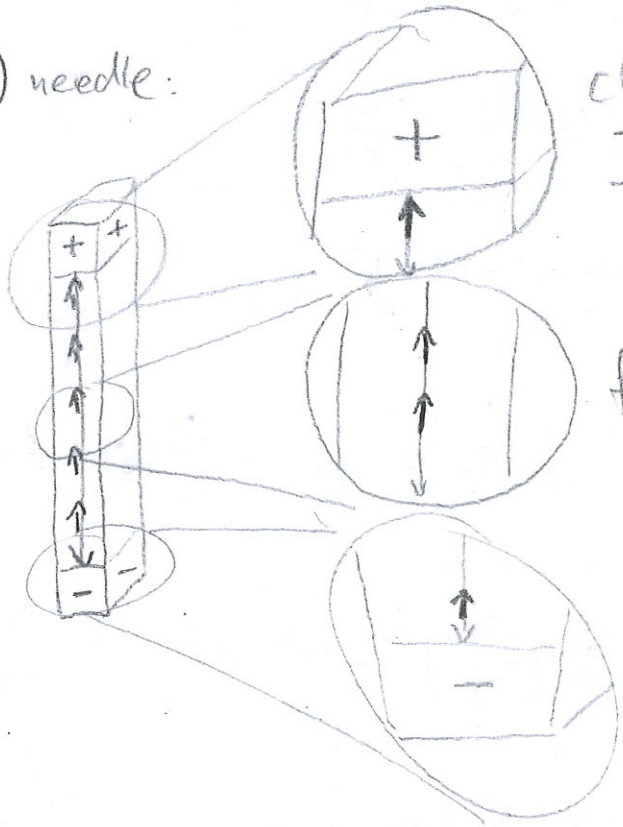
a) needle:



close to charges Q_m
→ r small
→ $H_d \propto \frac{Q_m}{r^2}$ is large

far away from charges Q_m
→ $r \to \infty$

close to charges $-Q_m$
→ $(a-r)$ small
→ $H_d \propto \frac{Q_m}{(a-r)^2}$ is large

H_d large

H_d negligible

H_d large

⇒ almost all of the mag. moments $M \cdot V$ are in a negligibly small field H_d

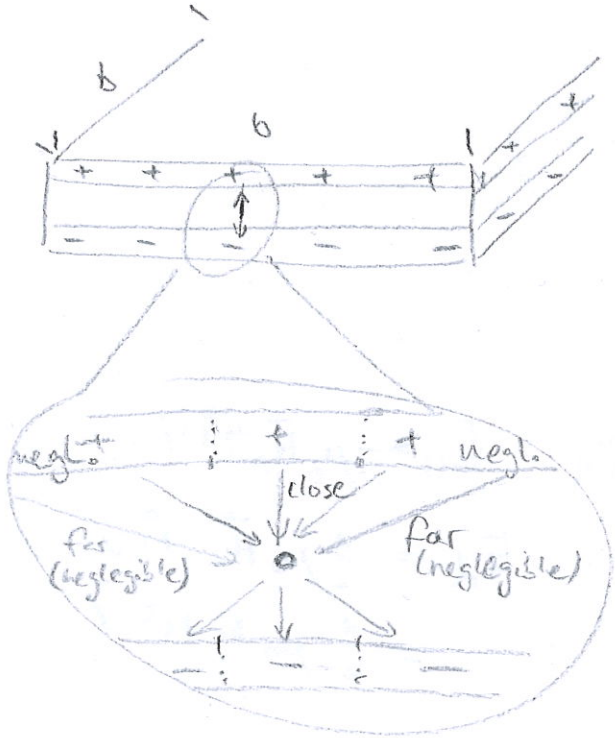
$\Rightarrow E_d = \int \mu_0 H_d M \, dV \ll \mu_0 H_d M V \xrightarrow{\lim} 0$

only small fraction of needle

⇒ for very elongated needles:

$$\boxed{N \to 0}$$

b)



⇒ field H_d that the mag. moment M experiences is due to mag. charges Q_m directly above and below it

⇒ total charge is $Q_m \propto \frac{1}{2} M \cdot b^2$
i.e. charge directly above and below is $Q_m / b^2 \propto \frac{1}{2} M$

⇒ $E_d = \int \mu_0 H_d M \, dV = \frac{1}{2} \int \mu_0 M \cdot M \, dV = \frac{1}{2} \mu_0 M \cdot M V$

⇒ $\boxed{N = 1}$